Appendix: The Proof of Theorem 2

1 Theorem

We review the Theorem 2 in our paper:

**Theorem 2.** With given $B$ and $C$, if $\rho > \max\{\rho_1, \rho_2, \rho_3\}$:

\[
\rho_1 = 6N\tau \left( \|B\|_F^4 + \|C\|_F^4 \right) / \left( \|B\|_F^2 + \|C\|_F^2 \right) \\
\rho_2 = 2 \|E\|_F^2 = \frac{6}{\rho_2} \left( 16N + N\tau \left( \|B\|_F^2 + \|C\|_F^2 \right) \right)^2 \\
\rho_3 = \|B\|_F^2 + \|C\|_F^2 + \|\mathcal{R}_p(E) + \mathcal{R}_q(E)\|_F^2
\]

We can claim that:

- The equality constraint on the auxiliary matrix is satisfied in the limit, i.e., $\lim_{t \to \infty} \|H(t) - V(t)\|_F^2 = 0$.
- The sequence $\{H(t), V(t), \Lambda(t)\}$ generated by the NS-Alternating algorithm is bounded, and any limit point of the sequence is a KKT point of problem (6) of our paper.

2 Proof

We first give the lemmas of our theorem and then present the derivation of these lemmas. According to the study [Hong et al., 2016], to give Theorem 2, we need to ensure that:

1. The size of the successive difference of the multipliers is bounded by that of the successive difference of the primal variables. (2) The augmented Lagrangian is decreasing and lower bounded.

**Lemma 1.** We have a bounded successive difference of the multipliers, that is:

\[
\|\Lambda^{(t+1)} - \Lambda^{(t)}\|_F^2 \leq 3c_1 \cdot \|H^{(t+1)} - H^{(t)}\|_F^2 \\
+ 3c_2 \cdot \|V^{(t+1)} - V^{(t)}\|_F^2 \\
+ 3c_3 \cdot \|H^{(t+1)}(V^{(t+1)} - V^{(t)})\|_F^2
\]

where $c_1$, $c_2$, and $c_3$ are positive scalars:

\[
c_1 = \left( 16N + \tau N \left( \|B\|_F^2 + \|C\|_F^2 \right) \right)^2
\]

\[
c_2 = \|B\|_F^2 \cdot \|H^{(t)}(V^{(t)}B)^T - M\|_F^2 \\
+ \|C\|_F^2 \cdot \|H^{(t)}(V^{(t)}C)^T - S\|_F^2
\]

\[
c_3 = N\tau \cdot \left( \|B\|_F^2 + \|C\|_F^2 \right)
\]

**Lemma 2.** If the equations below are satisfied,

\[
\rho > \max\{\rho_1, \rho_2\}
\]

\[
\rho_1 = 6N\tau \left( \|B\|_F^4 + \|C\|_F^4 \right) / \left( \|B\|_F^2 + \|C\|_F^2 \right) \\
\rho_2 = 2 \|E\|_F^2 + \frac{6}{\rho_2} \left( 16N + N\tau \left( \|B\|_F^2 + \|C\|_F^2 \right) \right)^2 \\
\rho_3 = \|B\|_F^2 + \|C\|_F^2 + \|\mathcal{R}_p(E) + \mathcal{R}_q(E)\|_F^2
\]

we have positive scalars $c_1$, $c_2$, and $c_3$ so that:

\[
\mathcal{L}(H^{(t+1)}, V^{(t+1)}, \Lambda^{(t+1)}) - \mathcal{L}(X^{(t)}, V^{(t)}, \Lambda^{(t)}) \\
< -c_1 \|H^{(t+1)}(V^{(t+1)} - V^{(t)})\|_F^2 \\
- c_2 \|H^{(t+1)} - H^{(t)}\|_F^2 \\
- c_3 \|V^{(t+1)} - V^{(t)}\|_F^2 \\
- \rho \|V^{(t)} - V^{(t+1)}\|_F^2 \\
- \rho \|H^{(t)} - H^{(t+1)}\|_F^2 \\
+ \rho \|C\|_F^2 \cdot \|H^{(t)}(V^{(t)}C)^T - S\|_F^2
\]

**Lemma 3.** If the equation below is satisfied,

\[
\rho \geq \|B\|_F^2 + \|C\|_F^2 + 2 \|\mathcal{R}_p(E) + \mathcal{R}_q(E)\|_F^2
\]

we have lower bound of 0, that is,

\[
\mathcal{L}(H^{(t+1)}, V^{(t+1)}, \Lambda^{(t+1)}) \geq 0
\]

Note that, we use $X_t$ instead of $\tilde{X}_t$ to denote the block diagonal matrix in this material. For the ease of derivation, we introduce elementary matrices, $E_1$ and $E_2$, to characterize the rotating operator $\mathcal{R}_p$ and $\mathcal{R}_q$, respectively.

**Derivation of Lemma 1:**

We first give the optimal condition of $\Lambda$ as follows:

\[
\left( H(BV^{(t+1)} - M) \right) \cdot V^{(t+1)}B^T \\
+ \left( HCV^{(t+1)}T - S \right) \cdot V^{(t+1)}C^T \\
+ \rho \left( H - V^{(t+1)} - \Lambda^{(t)} \right)/\rho \left( H - V^{(t+1)} - \Lambda^{(t)} \right)
\]

Together with the updating rule of $\Lambda$, we obtain:

\[
\Lambda^{(t+1)} = \left( H^{(t+1)BV^{(t+1)}T - M} \right) \cdot V^{(t+1)}B^T \\
+ \left( H^{(t+1)CV^{(t+1)}T - S} \right) \cdot V^{(t+1)}C^T \\
+ \left( H^{(t+1)} - V^{(t+1)} + \Lambda^{(t+1)} \right)
\]
Then, the successive difference of $\lambda$ is given below:

$$
\Lambda^{(t+1)} - \Lambda^{(t)} = 
\begin{align*}
&= \left( H^{(t+1)} BV^{(t+1)^T} - M \right) \cdot V^{(t+1)} B^T \\
&\quad + \left( H^{(t+1)} CV^{(t+1)^T} - S \right) \cdot V^{(t+1)} C^T \\
&\quad + 2 \left( E^T_1 E_1 - E^T_2 E_2 - E^T_2 E_1 + E^T_1 E_2 \right) \left( H^{(t+1)} - H^{(t)} \right) \\
&\quad + \left( H^{(t)} BV^{(t)^T} - M \right) \cdot V^{(t)} B^T \\
&\quad + \left( H^{(t)} CV^{(t)^T} - S \right) \cdot V^{(t)} C^T \\
&\quad + H^{(t+1)} BV^{(t+1)^T} V^{(t+1)} B^T - H^{(t)} BV^{(t)^T} V^{(t)} B^T \\
&\quad + H^{(t+1)} CV^{(t+1)^T} V^{(t+1)} C^T - H^{(t)} CV^{(t)^T} V^{(t)} C^T \\
&\quad - M \left( V^{(t+1)} B^T - V^{(t)} B^T \right) - S \left( V^{(t+1)} C^T - V^{(t)} C^T \right) \\
&\quad + 2 \left( E^T_1 E_1 - E^T_2 E_2 - E^T_2 E_1 + E^T_1 E_2 \right) \left( H^{(t+1)} - H^{(t)} \right) \\
&\quad = \left( H^{(t+1)} - H^{(t)} \right) \left( V^{(t+1)} B^T \right)^T \left( V^{(t+1)} B^T \right) \\
&\quad + H^{(t)} \left[ \left( V^{(t+1)} B^T \right)^T \left( V^{(t+1)} B^T \right) \right] \\
&\quad + \left( H^{(t+1)} - H^{(t)} \right) \left( V^{(t+1)} C^T \right)^T \left( V^{(t+1)} C^T \right) \\
&\quad + M \left( V^{(t+1)} B^T - V^{(t)} B^T \right) - S \left( V^{(t+1)} C^T - V^{(t)} C^T \right) \\
&\quad + 2 \left( E^T_1 E_1 - E^T_2 E_2 - E^T_2 E_1 + E^T_1 E_2 \right) \left( H^{(t+1)} - H^{(t)} \right) \\
&\quad = \left( H^{(t)} \left( V^{(t+1)} B^T \right)^T \right. \\
&\quad \left. - M \left( V^{(t+1)} B^T - V^{(t)} B^T \right) \\
&\quad - \left( H^{(t)} \left( V^{(t+1)} C^T \right)^T \right. \\
&\quad \left. - S \left( V^{(t+1)} C^T - V^{(t)} C^T \right) \\
&\quad + M \left( V^{(t+1)} B^T - V^{(t)} B^T \right) - S \left( V^{(t+1)} C^T - V^{(t)} C^T \right) \\
&\quad + 2 \left( E^T_1 E_1 - E^T_2 E_2 - E^T_2 E_1 + E^T_1 E_2 \right) \left( H^{(t+1)} - H^{(t)} \right) \\
&\quad + H^{(t)} \left( V^{(t+1)} B^T - V^{(t)} B^T \right)^T \left( V^{(t+1)} B^T \right) \\
&\quad + H^{(t)} \left( V^{(t+1)} C^T - V^{(t)} C^T \right)^T \left( V^{(t+1)} C^T \right)
\end{align*}
$$

Using triangle inequality, we obtain:

$$
\| |A^{(t+1)} - A^{(t)}| \|_F \\
\leq \| H^{(t+1)} - H^{(t)} \|_F \| \left( (V^{(t+1)} B^T)^T \left( V^{(t+1)} B^T \right) \right) \\
+ 2 \left( E^T_1 E_1 - E^T_2 E_2 - E^T_2 E_1 + E^T_1 E_2 \right) \left( H^{(t+1)} - H^{(t)} \right) \|_F \\
+ \| H^{(t)} \left( V^{(t)} C^T \right)^T \left( V^{(t+1)} B^T \right) \|_F \\
+ \| \left( V^{(t+1)} B^T - V^{(t)} B^T \right) \|_F \| H^{(t)} \left( V^{(t+1)} B^T - V^{(t)} B^T \right) \|_F \\
+ \| \left( V^{(t+1)} C^T - V^{(t)} C^T \right) \|_F \| H^{(t)} \left( V^{(t+1)} C^T - V^{(t)} C^T \right) \|_F \\
+ \| H^{(t)} \left( V^{(t+1)} B^T - V^{(t)} B^T \right)^T \|_F \| H^{(t+1)} B^T \|_F \\
+ \| H^{(t)} \left( V^{(t+1)} C^T - V^{(t)} C^T \right)^T \|_F \| H^{(t+1)} C^T \|_F
$$

Given $\| V \|_F \leq \sqrt{N_T}$, finally, we obtain:

$$
\| |A^{(t+1)} - A^{(t)}| \|_F \\
\leq 3 \left( 16N + \tau N \left| \left( (B_D)^2 + (C_D)^2 \right) \right| \right) \left( H^{(t+1)} - H^{(t)} \right) \|_F \\
+ 3 \left( (B_D)^2 \| H^{(t)} \left( V^{(t)} B^T \right) \|_F^2 - M \|_F^2 \\
+ 3 \left( (C_D)^2 \| H^{(t)} \left( V^{(t)} C^T \right) \|_F^2 - S \|_F^2 \\
+ 3N_T \left( (B_D)^2 + (C_D)^2 \right) \left( H^{(t+1)} - H^{(t)} \right) \|_F^2 \\
+ 3N_T \left( (B_D)^2 + (C_D)^2 \right) \left( H^{(t+1)} - H^{(t)} \right) \|_F^2 \\
+ \left( V^{(t+1)} - V^{(t)} \right) \|_F^2
$$

Derivation of Lemma 2:

First, we let

$$
\begin{align*}
A &\triangleq \mathcal{L} \left( H^{(t)}, V^{(t+1)}, A^{(t)} \right) - \mathcal{L} \left( H^{(t)}, V^{(t)}, A^{(t)} \right) \\
B &\triangleq \mathcal{L} \left( H^{(t+1)}, V^{(t+1)}, A^{(t)} \right) - \mathcal{L} \left( H^{(t)}, V^{(t+1)}, A^{(t)} \right) \\
C &\triangleq \mathcal{L} \left( H^{(t+1)}, V^{(t+1)}, A^{(t+1)} \right) - \mathcal{L} \left( H^{(t+1)}, V^{(t+1)}, A^{(t)} \right) \\
\end{align*}
$$

where

$$
\begin{align*}
\mathcal{L} \left( H^{(t)}, V^{(t)}, A^{(t)} \right) &\triangleq \frac{1}{2} \left( H^{(t)} BV^T - M \right) \|_F^2 + \frac{1}{2} \left( H^{(t)} CV^T - S \right) \|_F^2 \\
+ \frac{\rho}{2} \| \left( H^{(t)} - V + A^{(t)} / \rho \right) \|_F^2 + \| E_1 H - E_2 H \|_F^2 \\
+ \frac{\beta^{(t)}}{2} \| V - \lambda^{(t)} \|_F^2
\end{align*}
$$
Then, we have

\[ \mathcal{L} \left( \mathbf{H}^{(t+1)}, \mathbf{V}^{(t+1)}, \mathbf{A}^{(t+1)} \right) - \mathcal{L} \left( \mathbf{H}^{(t)}, \mathbf{V}^{(t+1)}, \mathbf{A}^{(t)} \right) = A + B + C \leq A + B + C \quad (9) \]

Specifically,

\[ \hat{A} = \frac{1}{2} \| \mathbf{H}^{(t)} \mathbf{B} \mathbf{V}^{(t+1)^T} - \mathbf{M} \|_F^2 + \frac{1}{2} \| \mathbf{H}^{(t)} \mathbf{B} \mathbf{V}^{(t+1)^T} - \mathbf{M} \|_F^2 \\
+ \frac{1}{2} \| \mathbf{H}^{(t)} \mathbf{C} \mathbf{V}^{(t+1)^T} - \mathbf{S} \|_F^2 + \frac{1}{2} \| \mathbf{H}^{(t)} \mathbf{C} \mathbf{V}^{(t+1)^T} - \mathbf{S} \|_F^2 \\
+ \frac{\rho}{2} \| \mathbf{H}^{(t)} - \mathbf{V}^{(t+1)} + \frac{\Lambda^{(t)}}{\rho} \|_F^2 - \frac{\rho}{2} \| \mathbf{H}^{(t)} - \mathbf{V}^{(t)} + \frac{\Lambda^{(t)}}{\rho} \|_F^2 \\
+ \frac{\beta^{(t)}}{2} \| \mathbf{V} - \mathbf{V}^{(t)} \|_F^2 \\
\leq \frac{\rho}{2} \| \mathbf{V}^{(t+1)} - \mathbf{V}^{(t)} \|_F^2 + \frac{\beta^{(t)}}{2} \| \mathbf{V}^{(t+1)} - \mathbf{V}^{(t)} \|_F^2 \\
\leq -\frac{1}{2} \| \mathbf{V}^{(t+1)} - \mathbf{V}^{(t)} \|_F^2 \quad (10) \]

where (a) is the Taylor expansion and (b) is optimal condition.

Similarly,

\[ B = \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{B} \mathbf{V}^{(t+1)^T} - \mathbf{M} \|_F^2 + \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{B} \mathbf{V}^{(t+1)^T} - \mathbf{M} \|_F^2 \\
+ \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{C} \mathbf{V}^{(t+1)^T} - \mathbf{S} \|_F^2 + \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{C} \mathbf{V}^{(t+1)^T} - \mathbf{S} \|_F^2 \\
+ \frac{\rho}{2} \| \mathbf{H}^{(t)} - \mathbf{V}^{(t+1)} + \frac{\Lambda^{(t)}}{\rho} \|_F^2 - \frac{\rho}{2} \| \mathbf{H}^{(t)} - \mathbf{V}^{(t)} + \frac{\Lambda^{(t)}}{\rho} \|_F^2 \\
\leq -\frac{1}{2} \| \mathbf{B} \mathbf{F} + \| \mathbf{C} \|_F^2 \| \mathbf{H}^{(t)} - \mathbf{V}^{(t)} \|_F^2 \\
\leq -\frac{\rho}{2} \| \mathbf{V}^{(t+1)} - \mathbf{V}^{(t)} \|_F^2 + \frac{\beta^{(t)}}{2} \| \mathbf{V}^{(t+1)} - \mathbf{V}^{(t)} \|_F^2 \quad (11) \]

and

\[ C = \frac{\rho}{2} \| \mathbf{V}^{(t+1)} - \mathbf{H}^{(t+1)} + \frac{\Lambda^{(t+1)}}{\rho} \|_F^2 - \frac{\rho}{2} \| \mathbf{V}^{(t+1)} - \mathbf{H}^{(t+1)} + \frac{\Lambda^{(t+1)}}{\rho} \|_F^2 \\
\leq \frac{1}{\rho} \| \mathbf{V}^{(t+1)} - \mathbf{V}^{(t)} \|_F^2 \quad (12) \]

Then, we need to incorporate the result of Lemma 1 into C.

Finally,

\[ \mathcal{L} \left( \mathbf{H}^{(t+1)}, \mathbf{V}^{(t+1)}, \mathbf{A}^{(t+1)} \right) - \mathcal{L} \left( \mathbf{H}^{(t)}, \mathbf{V}^{(t)}, \mathbf{A}^{(t)} \right) \]

\[ \leq \hat{A} + B + C \]

\[ \leq -\frac{1}{2} \| \mathbf{B} \mathbf{F} + \| \mathbf{C} \|_F^2 \| \mathbf{H}^{(t)} - \mathbf{V}^{(t)} \|_F^2 \quad (13) \]

where

\[ c_1 = \frac{\rho}{2} + \| \mathbf{E}_1 - \mathbf{E}_2 \|_F^2 - \frac{3}{\rho} (16N + N\tau \| \mathbf{B} \mathbf{F} + \| \mathbf{C} \|_F^2)^2 \]

\[ c_2 = \frac{\rho}{2} + \| \mathbf{E}_1 - \mathbf{E}_2 \|_F^2 - \frac{3}{\rho} \| \mathbf{H}^{(t)} - \mathbf{V}^{(t)} \|_F^2 \quad (14) \]

Let \( c_i > 0 \), \( i = \{1, 2, 3, 4\} \), we have Lemma 2.

Derivation of Lemma 3:

\[ \mathcal{L} \left( \mathbf{H}^{(t+1)}, \mathbf{V}^{(t+1)}, \mathbf{A}^{(t+1)} \right) \]

\[ = \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{B} \mathbf{V}^{(t+1)^T} - \mathbf{M} \|_F^2 + \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{C} \mathbf{V}^{(t+1)^T} - \mathbf{S} \|_F^2 + \| \mathbf{E}_1 \mathbf{H}^{(t+1)} - \mathbf{E}_2 \mathbf{H}^{(t+1)} \|_F^2 + \frac{\rho}{2} \| \mathbf{V}^{(t+1)} - \mathbf{H}^{(t+1)} + \frac{\Lambda^{(t+1)}}{\rho} \|_F^2 \quad (15) \]

\[ = \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{B} \mathbf{V}^{(t+1)^T} - \mathbf{M} \|_F^2 + \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{C} \mathbf{V}^{(t+1)^T} - \mathbf{S} \|_F^2 + \| \mathbf{E}_1 \mathbf{H}^{(t+1)} - \mathbf{E}_2 \mathbf{H}^{(t+1)} \|_F^2 + \frac{\rho}{2} \| \mathbf{V}^{(t+1)} - \mathbf{H}^{(t+1)} + \frac{\Lambda^{(t+1)}}{\rho} \|_F^2 \quad (15) \]

\[ = \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{B} \mathbf{V}^{(t+1)^T} - \mathbf{M} \|_F^2 + \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{C} \mathbf{V}^{(t+1)^T} - \mathbf{S} \|_F^2 + \| \mathbf{E}_1 \mathbf{H}^{(t+1)} - \mathbf{E}_2 \mathbf{H}^{(t+1)} \|_F^2 + \frac{\rho}{2} \| \mathbf{V}^{(t+1)} - \mathbf{H}^{(t+1)} + \frac{\Lambda^{(t+1)}}{\rho} \|_F^2 \quad (15) \]

\[ = \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{B} \mathbf{V}^{(t+1)^T} - \mathbf{M} \|_F^2 + \frac{1}{2} \| \mathbf{H}^{(t+1)} \mathbf{C} \mathbf{V}^{(t+1)^T} - \mathbf{S} \|_F^2 + \| \mathbf{E}_1 \mathbf{H}^{(t+1)} - \mathbf{E}_2 \mathbf{H}^{(t+1)} \|_F^2 + \frac{\rho}{2} \| \mathbf{V}^{(t+1)} - \mathbf{H}^{(t+1)} + \frac{\Lambda^{(t+1)}}{\rho} \|_F^2 \quad (15) \]
First, we note that,
\[
0 \leq \left\| (H^{(t+1)} - V^{(t+1)}) BV^{(t+1)^T} + (H^{(t+1)}BV^{(t+1)^T} - M) \right\|_F^2
\]
\[
= \left\| (H^{(t+1)} - V^{(t+1)}) BV^{(t+1)^T} \right\|_F^2 + \left\| (H^{(t+1)}BV^{(t+1)^T} - M) \right\|_F^2
\]
\[
+ 2 \left\langle BV^{(t+1)^T} (H^{(t+1)} - V^{(t+1)}) , (H^{(t+1)}BV^{(t+1)^T} - M) \right\rangle
\]
\[
= \left\| (H^{(t+1)} - V^{(t+1)}) BV^{(t+1)^T} \right\|_F^2 + \left\| (H^{(t+1)}BV^{(t+1)^T} - M) \right\|_F^2
\]
\[
+ 2 \left\langle (H^{(t+1)} - V^{(t+1)}) , (H^{(t+1)}BV^{(t+1)^T} - M) V^{(t+1)^T} \right\rangle,
\]
where we have
\[
(Y^T (X - Y), XY - Z) = (X - Y, (XY - Z)Y).
\]
That is,
\[
\left\| (H^{(t+1)}BV^{(t+1)^T} - M) \right\|_F^2
\]
\[
+ 2 \left\langle (H^{(t+1)} - V^{(t+1)}) , (H^{(t+1)}BV^{(t+1)^T} - M) \right\rangle
\]
\[
\leq - \left\| (H^{(t+1)} - V^{(t+1)}) BV^{(t+1)^T} \right\|_F^2
\]
\[
- \left\| B \right\|_F^2 N \tau \left\| (H^{(t+1)} - V^{(t+1)}) \right\|_F^2.
\]
Therefore, we obtain
\[
\frac{1}{2} \left\| (H^{(t+1)}BV^{(t+1)^T} - M) \right\|_F^2
\]
\[
+ 2 \left\langle (H^{(t+1)} - V^{(t+1)}) , (H^{(t+1)}BV^{(t+1)^T} - M) \right\rangle
\]
\[
\leq - \frac{1}{2} \left\| B \right\|_F^2 N \tau \left\| (H^{(t+1)} - V^{(t+1)}) \right\|_F^2.
\]
Second, we similarly derive the inequality as follows:
\[
\frac{1}{2} \left\| (H^{(t+1)}CV^{(t+1)^T} - S) \right\|_F^2
\]
\[
+ 2 \left\langle (H^{(t+1)} - V^{(t+1)}) , (H^{(t+1)}CV^{(t+1)^T} - S) \right\rangle
\]
\[
\leq - \frac{1}{2} \left\| C \right\|_F^2 N \tau \left\| (H^{(t+1)} - V^{(t+1)}) \right\|_F^2.
\]
Third, we first derive equations as follows:
\[
2 \left\langle (H^{(t+1)} - V^{(t+1)}) , (E_1 - E_2)^T (E_1 - E_2) H^{(t+1)} \right\rangle
\]
\[
+ \left\| E_1 H^{(t+1)} - E_2 H^{(t+1)} \right\|_F^2
\]
\[
= 2 \left\langle (H^{(t+1)} - V^{(t+1)}) , (E_1 - E_2)^T (E_1 - E_2) H^{(t+1)} \right\rangle
\]
\[
+ \left\| (E_1 - E_2) H^{(t+1)} \right\|_F^2
\]
\[
= 2 \left\langle (H^{(t+1)} - V^{(t+1)}) , (E_1 - E_2)^T (E_1 - E_2) H^{(t+1)} \right\rangle
\]
\[
+ \left\| (E_1 - E_2) H^{(t+1)} \right\|_F^2.
\]
Note that,
\[
0 \leq \left\| (H^{(t+1)} - V^{(t+1)}) (E_1 - E_2) - (E_1 - E_2) H^{(t+1)} \right\|_F^2
\]
\[
= \left\| (H^{(t+1)} - V^{(t+1)}) (E_1 - E_2) \right\|_F^2 + \left\| (E_1 - E_2) H^{(t+1)} \right\|_F^2
\]
\[
+ 2 \left\langle (H^{(t+1)} - V^{(t+1)}) (E_1 - E_2) , (E_1 - E_2) H^{(t+1)} \right\rangle.
\]
That is,
\[
\left\| (E_1 - E_2) H^{(t+1)} \right\|_F^2
\]
\[
+ 2 \left\langle (H^{(t+1)} - V^{(t+1)}) (E_1 - E_2) , (E_1 - E_2) H^{(t+1)} \right\rangle
\]
\[
\geq - \left\| (E_1 - E_2) \right\|_F^2 \cdot \left\| (H^{(t+1)} - V^{(t+1)}) \right\|_F^2.
\]
Therefore, we obtain
\[
2 \left\langle (H^{(t+1)} - V^{(t+1)}) (E_1 - E_2) , (E_1 - E_2) H^{(t+1)} \right\rangle
\]
\[
+ \left\| (E_1 - E_2) H^{(t+1)} \right\|_F^2
\]
\[
\geq - \left\| (E_1 - E_2) \right\|_F^2 \cdot \left\| (H^{(t+1)} - V^{(t+1)}) \right\|_F^2.
\]
Finally, we obtain:
\[
\mathcal{L} (H^{(t+1)} , V^{(t+1)} , A^{(t+1)}) = c \cdot \left\| (H^{(t+1)} - V^{(t+1)}) \right\|_F^2
\]
\[
c = \rho - \frac{1}{2} \left\| B \right\|_F^2 N \tau - \frac{1}{2} \left\| C \right\|_F^2 N \tau - \left\| E_1 - E_2 \right\|_F^2
\]
Let $c > 0$, we have Lemma 3.

With Lemma 1-3 and the optimal condition, we can claim that Theorem 2 holds. Q.E.D.

**References**